Fluid Equations.

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Different Physics...different laws

- Mechanics.
- Thermodynamics.
- Electromagnetism.

Mathematical modeling.

- Nature is described by a finite number of equations.

- The difference between reality and the computed results:
  - Some minor physical effects are neglected.
  - Numerical approximations.(Iterative processes, Roundoff errors, etc...)
  - Geometrical approximations.

Mechanics.

- Newton's Laws
  - First law: The velocity of a body remains constant unless the body is acted upon by an external force.
  - Second law: The variation of a body momentum in time is parallel and directly proportional to the net force F and inversely proportional to the mass m.

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

  - Third law: The mutual forces of action and reaction between two bodies are equal, opposite and collinear.

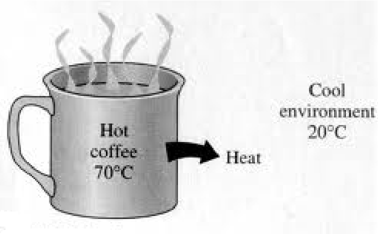
And God said...

$$
\begin{aligned}
\nabla \times \overrightarrow{E} &= -\mu \frac{\partial \overrightarrow{H}}{\partial t} \\
\nabla \times \overrightarrow{H} &= \overrightarrow{J} + \varepsilon \frac{\partial \overrightarrow{E}}{\partial t} \\
\nabla \cdot \overrightarrow{D} &= \rho \\
\nabla \cdot \overrightarrow{B} &= 0
\end{aligned}
$$

and there was light.

Thermodynamics

- First law of thermodynamics, about the conservation of energy:
  - Energy is neither created nor destroyed.
  - There is no free lunch.
- Second law of thermodynamics, about entropy:
  - In an isolated system, the entropy never decreases.
  - Heat cannot spontaneously flow from a colder location to a hotter area – work is required to achieve this.



Heat flows in the direction of decreasing temperature.

Fluid Mechanics.

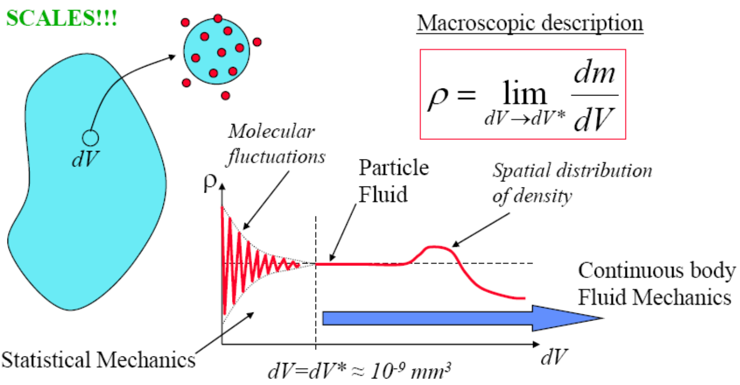
- Particular case of classical Mechanics.
- Core ideas coming from Thermodynamics.
- Extension of Newton's law to a complex system.

Basic hypothesis.

- Continuum hypothesis.
- Mass conservation.
- Second Newton's law. $\mathbf{F} = \frac{d\mathbf{p}}{dt}$
- First Principle of Thermodynamics: Energy conservation.

Continuum hypothesis.

- Fluids have a molecular nature. $V = 10^{-9} mm^3$ air in NC $\Rightarrow 3 \cdot 10^7$molecules



Fluid particle.

- Mathematical concept.
- What size?
  - Large enough to contain many molecules.
  - Small enough to allow the use of the differential calculus.
- Hypothesis: Local thermodynamic equilibrium.
  - Random free walk $\lambda \ll$ Problem dimensions.
  - Average time between molecular collisions $\ll$ rate of change of the fluid variables.
- Conclusion: Finite volume (fluid particle) is defined by one velocity $\mathbf{v}$, pressure p, density ρ, Temperature T, etc...

How to work with fluids.

- Fundamental laws:
  - Mass conservation.
  - Momentum conservation.
  - Energy conservation.
- Extra information:
  - Equations of state.
  - Boundary conditions.

$\Rightarrow$ System of non-linear partial differential equations.

- Difficult analytical solution
- Expensive and difficult experiments.

$\Rightarrow$ **Numerical solution.**

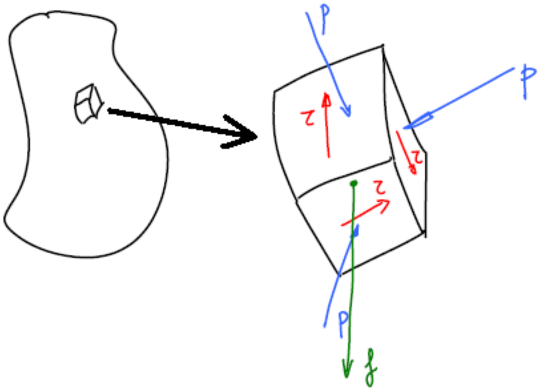
Fluid equations for a fluid volume.

- M total mass of our fluid volume.
- $\mathbf{P}$ total momentum of our fluid volume.
- $\mathbf{F}$ total force that our fluid is experiencing.
- E total energy that our fluid contains.
- Q total heat that our fluid is transferring.
- W total work that our fluid is performing.

$$\frac{dM}{dt} = 0$$
$$\frac{d\mathbf{P}}{dt} = \mathbf{F}$$
$$\frac{dE}{dt} = Q + W$$

Forces over a fluid particle.

- External forces f. Examples: gravity, electromagnetic, inertial, etc...
- Friction forces. The particles experience the force by physical contact. Examples: viscous friction and pressure.



Fluid equations of a fluid particle: Navier-Stokes equations. And God said again:

- Incompressible fluid.
- Newtonian fluid.
- Only mechanical properties will be considered, thermal effects will be neglected.

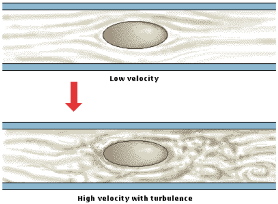
$$\begin{aligned}\nabla \cdot \mathbf{v} &= 0\\ \rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} &= -\nabla p + \rho \mathbf{f} + \nabla \tau\end{aligned}$$

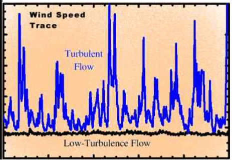
- $\mathbf{f}$ external volumetric forces: gravity, electromagnetic, inertial, etc...
- τ viscous stress tensor.

Vocabulary.

- Compressibility: possibility of density changes.
  - Liquids always incompressible $\rho = const.$
  - Gases are compressible under some hypothesis.

- Laminar and turbulent.





Turbulent flow = Averaged flow + Fluctuation.

- Stationary $\frac{\partial}{\partial t} = 0$ and time dependant $\frac{\partial}{\partial t} \neq 0$.

- Unidirectional: One velocity component.

Navier-Stokes Equations. Incompressible without thermal effects $\mu = cons$.

$$\nabla \cdot \mathbf{v} = 0$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \rho \mathbf{f} + \mu \nabla^2 \mathbf{v}$$

Navier-Stokes Equations. Particularities

- Continuum flows, such as flows in porous media, unsteady separated and turbulent flows, are inherently **multiscale** due to the range of scales that govern the underlying physical phenomena.

- The continuum assumption fails in flow regions containing contact lines and shocks, and suitable molecular descriptions become necessary.